n-Valued Refined Neutrosophic Soft Set Theory

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***Abstract*—In this paper as a generalization of neutrosophic soft set we introduce the concept of n-valued refined neutrosophic soft set and study some of its properties. We also, define its basic operations, complement, union intersection, "AND" and "OR" and study their properties.**

I. Introduction

Neutrosophic set was introduced in 1995 by Florentin Smarandache, who coined the words ĎneutrosophyŤ and its derivative ĎneutrosophicŤ. Smarandache in [1] and [2] introduced the concept of neutrosophic set which is a mathematical tool for handling problems involving imprecise, indeterminacy and inconsistent data. In 1995 [3] Smarandache also showed that the neutrosophic set is a generalization of the intuitionistic fuzzy sets. The neutrosophic numerical components (t,i,f) are crisp numbers, intervals, or in general subsets of the unitary standard or nonstandard unit interval. Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. This theory considers every notion or idea $\langle A\rangle$ together with its opposite or negation $\langle antiA\rangle$ and with their spectrum of neutralities $\langle neutA\rangle$ in between them (i.e. notions or ideas supporting neither $\langle A\rangle$ nor $\langle antiA\rangle$). In 2015 Smarandache [4] presented a short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of n-symbol or numerical valued logic. He showed generalizations of 2-valued Boolean logic to fuzzy logic, also from the KleeneŠs and LukasiewiczŠ 3-symbol valued logics or BelnapŠs 4-symbol valued logic to the most general n-symbol or numerical valued refined neutrosophic logic. Also he gave a generalizations for n-valued refined neutrosophic set. In 2015 Agboola [5] developed refined neutrosophic algebraic structures by studding refined neutrosophic group and he presented some of its elementary properties. Broumi et. al. in [6] defined the concept of n-valued interval neutrosophic sets and introduced the basic operations of this concept such as; union, intersection, addition, multiplication, scalar multiplication, scalar division, truth favorite and false-favorite. In this paper they also some distances between n-valued interval neutrosophic sets are proposed . Also, they proposed an efficient approach for group multi-criteria decision making based on n-valued interval neutrosophic sets and give an application of n-valued interval neutrosophic sets in medical diagnosis problem. Smarandache in 2015 [7] gave a short history about: the neutrosophic set, neutrosophic numerical components and neutrosophic literal components, neutrosophic numbers, neutrosophic intervals, neutrosophic dual number, neutrosophic special dual number, neutrosophic special quasi dual number, neutrosophic linguistic number, neutrosophic linguistic interval-style number, neutrosophic hypercomplex numbers of dimension n, and elementary neutrosophic algebraic structures. He also gave their generalizations to refined neutrosophic set, respectively refined neutrosophic numerical and literal components, then refined neutrosophic numbers and refined neutrosophic algebraic structures, and set-style neutrosophic numbers. Broumi and Smarandache in 2014 [8] proposed the cosine similarity measure of neutrosophic refined (multi-) sets where the cosine similarity measure of neutrosophic refined sets is the extension of improved cosine similarity measure of single valued neutrosophic. They also presented the application of medical diagnosis using this cosine similarity measure of neutrosophic refined set. In 1999, Molodtsov [9] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. Maji et. al. [10] in 2003 studied soft set and gave some operations related to this theory. As a combination of neutrosophic set and soft set Maji [11] introduced neutrosophic soft set, established its application in decision making. In 2013 Said and Smarandache [12] defined the concept of intuitionistic neutrosophic soft set and introduced some operations on intuitionistic neutrosophic soft set and some properties of this concept have been established. Mehmet et. al. in 2015 [13] introduced the concept of neutrosophic soft expert set they also defined its basic operations, namely complement, union, intersection, AND and OR, and studied some of their properties and gave an application of this concept in a decision- making problem. In this paper firstly, we present a short history of logics: from particular cases of 2-symbol or numerical valued logic to the general case of n-symbol or numerical valued logic as presented in [4], [14], [7]. As a generalization of neutrosophic soft set we introduce the concept of n-valued refined neutrosophic soft set and study some of its properties. We also, define its basic operations, complement, union intersection, "AND" and "OR" and study their properties.

II. Preliminary

In this section we recall some definitions and properties regarding neutrosophic set theory, soft set theory time-fuzzy soft set and neutrosophic soft set theory required in this paper.

Definition 1. *[3] A neutrosophic set A on the universe of discourse X is defined as*

$$A = \{< x; T_A(x); I_A(x); F_A(x) >; x \in X\}$$

where T; I; $F : X \to]^{-}0; 1^{+}[$ and

$$^{-}0 \leqslant T_A(x) + I_A(x) + F_A(x) \leq 3^{+}.$$

Molodtsov defined soft set in the following way. Let U be a universe and E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subseteq E$.

Definition 2. *[9] A pair (F,A) is called a* soft set *over U, where F is a mapping*

$$F : A \to P(U).$$

In other words, a soft set over U is a parameterized family of subsets of the universe U. For $\varepsilon \in A, F(\varepsilon)$ may be considered as the set of ε-approximate elements of the soft set (F,A).

Definition 3. *[11] Let U be an initial universe set and E be a set of parameters. Consider $A \subset E$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F,A) is termed to be the soft neutrosophic set over U, where F is a mapping given by $F : A \to P(U)$.*

Definition 4. *[11] Let (F,A) and (G,B) be two neutrosophic soft sets over the common universe U. (F,A) is said to be neutrosophic soft subset of (G,B) if $A \subset B$; and $T_F(e)(x) \leq T_G(e)(x)$; $I_F(e)(x) \leq I_G(e)(x)$; $F_F(e)(x) \geq F_G(e)(x)$; $\forall e \in A; x \in U$. We denote it by $(F,A) \subseteq (G,B)$. (F,A) is said to be neutrosophic soft super set of (G,B) if (G,B) is a neutrosophic soft subset of (F,A). We denote it by $(F,A) \supseteq (G,B)$.*

Definition 5. *[11] The complement of a neutrosophic soft set (F,A) denoted by $(F;A)^c$ and is denoted as $(F,A)^c = (F^c, \rceil A)$; where $F^c : \rceil A \to P(U)$ is a mapping given by $F^c(\alpha)$ = neutrosophic soft complement with $T_{F^c(x)} = F_{F(x)}$, $I_{F^c(x)} = I_{F(x)}$ and $F_{F^c(x)} = T_{F(x)}$.*

Definition 6. *[11] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the union of (H,A) and (G,B) is denoted by '$(H,A) \cup (G,B)$' and is defined by $(H,A) \cup (G,B) = (K,C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:*

$$T_K(e)(m) = T_H(e)(m); \quad \text{if } e \in A - B;$$

$$= T_G(e)(m); \quad \text{if } e \in B - A;$$

$$= max(T_H(e)(m); T_G(e)(m)); \quad \text{if } e \in A \cap B.$$

$$I_K(e)(m) = I_H(e)(m); \quad \text{if } e \in A - B;$$

$$= I_G(e)(m); \quad \text{if } e \in B - A;$$

$$= \frac{I_H(e)(m) + I_G(e)(m))}{2}; \quad \text{if } e \in A \cap B.$$

$$F_K(e)(m) = F_H(e)(m); \quad \text{if } e \in A - B;$$

$$= F_G(e)(m); \quad \text{if } e \in B - A;$$

$$= min(F_H(e)(m); F_G(e)(m)); \quad \text{if } e \in A \cap B.$$

Definition 7. *[11] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the intersection of (H,A) and (G,B) is denoted by '$(H,A) \cup (G,B)$' and is defined by $(H,A) \cup (G,B) = (K,C)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K,C) are as follows:*

$$T_K(e)(m) = min(T_H(e)(m); T_G(e)(m));$$

$$I_K(e)(m) = \frac{I_H(e)(m) + I_G(e)(m))}{2} \text{ and}$$

$$F_K(e)(m) = max(F_H(e)(m); F_G(e)(m)); \quad \forall e \in C.$$

Definition 8. *[11] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the 'AND' operation on them is denoted by '$(H,A) \bigvee (G,B)$' and is defined by $(H,A) \bigvee (G,B) = (K, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(K, A \times B)$ are as follows:*

$$T_K(\alpha,\beta)(m) = min(T_H(\alpha)(m); T_G(\beta)(m));$$

$$I_K(\alpha,\beta)(m) = \frac{I_H(\alpha)(m) + I_G(\beta)(m))}{2} \text{ and}$$

$$F_K(\alpha,\beta)(m) = max(F_H(\alpha)(m); F_G(\beta)(m)); \quad \forall \alpha \in A, \forall \beta \in B.$$

Definition 9. *[11] Let (H,A) and (G,B) be two NSSs over the common universe U. Then the 'OR' operation on them is denoted by '$(H,A) \bigvee (G,B)$' and is defined by $(H,A) \bigvee (G,B) = (O, A \times B)$, where the truth-membership, indeterminacy-membership and falsity-membership of $(O, A \times B)$ are as follows:*

$$T_O(\alpha,\beta)(m) = max(T_H(\alpha)(m); T_G(\beta)(m));$$

$$I_O(\alpha,\beta)(m) = \frac{I_H(\alpha)(m) + I_G(\beta)(m))}{2} \text{ and}$$

$$F_O(\alpha,\beta)(m) = min(F_H(\alpha)(m); F_G(\beta)(m)); \quad \forall \alpha \in A, \forall \beta \in B.$$

A. n-Valued Refined Neutrosophic Logic

The Neutrosophic Logic value of a given proposition has the values T = truth, I = Indeterminacy, and F = falsehood. Smarandache have defined in 1995 two types of n-valued logic: symbolic and numerical:

- The n-Symbol-Valued Refined Neutrosophic Logic.
  In general: T can be split into many types of truths: $T_1; T_2; ...; T_p$, and I into many types of indeterminacies: $I_1; I_2; ...; I_r$, and F into many types of falsities: $F_1; F_2; ...; F_s$, where all $p; r; s \geq 1$ are integers, and $p+$

$r+s=n$. All subcomponents $T_j;I_k;F_l$ are symbols for $j\in\{1,2,...,p\}$, $k\in\{1,2,...,r\}$, and $l\in\{1,2,...,s\}$.

- The n-Numerical-Valued Refined Neutrosophic Logic. In the same way, but all subcomponents $T_j;I_k;F_l$ are not symbols, but subsets of $[0,1]$, for all $j\in\{1,2,...,p\}$, all $k\in\{1,2,...,r\}$, and all $l\in\{1,2,...,s\}$..

Remark 1. *In this paper we use the second type of n-valued logics which is: The n-Numerical-Valued Refined Neutrosophic Logic.*

III. n-Valued Refined Neutrosophic Soft Set

In this section we introduce the definitions of n-valued refined neutrosophic soft set, derive some properties and give some examples.

Definition 10. *Let U be an initial universe set and E be a set of parameters, $T_j=\{T_1,T_2,...,T_p\}$ be a set types of truths, $I_k=\{I_1,I_2,...,I_r\}$ be a set types of indeterminacies and $F_k=\{F_1,F_2,...,F_s\}$ be a set types of falsities and $n=p+r+s$ where all subcomponents $T_j;I_k;F_l$ subsets of $[0,1]$. Consider $A\subset E$. Let $P(U)$ denotes the set of all n-Valued refined neutrosophic sets of U. The collection (f_n,A) is termed to be the n-valued refined neutrosophic soft set over U, where f_n is a mapping given by $f_n:A\to P(U)$.*

From the above general definition we can get the following spacial cases:

Case 1. *(4-valued refined neutrosophic soft set)*

Example 1. *Let $U=\{u_1,u_2\}$ be a set of universe, $E=\{e_1,e_2,e_3\}$ a set of parameters. Let the Indeterminacy I is refined (split) as $Un=Unknown$, and $C=contradiction$. T, F, Un and C are subsets of $[0,1]$. Then, we get the following case of 4-Valued refined neutrosophic soft set:*

$$f_4(e_1)=\left\{\frac{u_1}{\langle 0.5;(0.2,0.3);0.4\rangle},\frac{u_2}{\langle 0.7;(0.1,0.5);0.5\rangle}\right\},$$

$$f_4(e_2)=\left\{\frac{u_1}{\langle 0.3;(0.3,0.4);0.5\rangle},\frac{u_2}{\langle 0.3;(0.2,0.3);0.4\rangle}\right\},$$

$$f_4(e_3)=\left\{\frac{u_1}{\langle 0.6;(0.3,0.1);0.2\rangle},\frac{u_2}{\langle 0.5;(0.1,0.2);0.2\rangle}\right\},$$

and we can write the 4-valued refined neutrosophic soft set (f_4,E) as consisting of the following collection of approximations:

$$\begin{aligned}(f_4,E)=\Bigg\{&\left(e_1,\left\{\frac{u_1}{\langle 0.5;(0.2,0.3);0.4\rangle},\frac{u_2}{\langle 0.7;(0.1,0.5);0.5\rangle}\right\}\right),\\ &\left(e_2,\left\{\frac{u_1}{\langle 0.3;(0.3,0.4);0.5\rangle},\frac{u_2}{\langle 0.3;(0.2,0.3);0.4\rangle}\right\}\right),\\ &\left(e_3,\left\{\frac{u_1}{\langle 0.6;(0.3,0.1);0.2\rangle},\frac{u_2}{\langle 0.5;(0.1,0.2);0.2\rangle}\right\}\right)\Bigg\}.\end{aligned}$$

Also we can represent the above set as shown in Table I.

Case 2. *(5-valued refined neutrosophic soft set)*

Example 2. *consider the example as in Case 1. Let the Indeterminacy I is refined (split) as $Un=Unknown$, $C=contradiction$ and $G=ignorance$. T, F, Un, C and G are subsets of $[0,1]$. Then, we get the following case of 5-Valued refined neutrosophic soft set:*

$$f_5(e_1)=\left\{\frac{u_1}{\langle 0.5;(0.2,0.3,0.4);0.4\rangle},\frac{u_2}{\langle 0.7;(0.1,0.5,0.3);0.5\rangle}\right\},$$

$$f_5(e_2)=\left\{\frac{u_1}{\langle 0.3;(0.3,0.4,0.5);0.5\rangle},\frac{u_2}{\langle 0.3;(0.2,0.3,0.4);0.4\rangle}\right\},$$

$$f_5(e_3)=\left\{\frac{u_1}{\langle 0.6;(0.3,0.1,0.2);0.2\rangle},\frac{u_2}{\langle 0.5;(0.1,0.2,0.1);0.2\rangle}\right\},$$

and we can write the 5-valued refined neutrosophic soft set (f_5,E) as consisting of the following collection of approximations:

$$\begin{aligned}(f_5,E)=\Bigg\{&\left(e_1,\left\{\frac{u_1}{\langle 0.5;(0.2,0.3,0.4);0.4\rangle},\frac{u_2}{\langle 0.7;(0.1,0.5,0.3);0.5\rangle}\right\}\right),\\ &\left(e_2,\left\{\frac{u_1}{\langle 0.3;(0.3,0.4,0.5);0.5\rangle},\frac{u_2}{\langle 0.3;(0.2,0.3,0.4);0.4\rangle}\right\}\right),\\ &\left(e_3,\left\{\frac{u_1}{\langle 0.6;(0.3,0.1,0.2);0.2\rangle},\frac{u_2}{\langle 0.5;(0.1,0.2,0.1);0.2\rangle}\right\}\right)\Bigg\}.\end{aligned}$$

Also we can represent the above set as shown in Table II.

Case 3. *(6-valued refined neutrosophic soft set)*

Example 3. *Consider the example as in Case 1. Let the truth T is refined (split) as $T_A=AbsoluteTruth$, $T_R=RelativeTruth$, indeterminacy I is refined (split) as $I_A=AbsoluteIndeterminacy$, $I_R=RelativeIndeterminacy$ and and the falsity F is refined (split) as $F_A=Absolutefalsity$, $I_R=Relativefalsity$. T_A, T_R, I_A, I_R F_A and F_R are subsets of $[0,1]$. Then, we get the following case of 6-Valued refined neutrosophic soft set:*

$$f_5(e_1)=\left\{\frac{u_1}{\langle(0.5,0.4);(0.2,0.3);(0.4,0.3)\rangle},\frac{u_2}{\langle(0.7,0.4);(0.1,0.5);(0.5,0.1)\rangle}\right\},$$

$$f_5(e_2)=\left\{\frac{u_1}{\langle(0.3,0.5);(0.3,0.4);(0.5,0.4)\rangle},\frac{u_2}{\langle(0.3,0.3);(0.2,0.3);(0.4,0.8)\rangle}\right\},$$

$$f_5(e_3)=\left\{\frac{u_1}{\langle(0.6,0.3);(0.3,0.1);(0.2,0.4)\rangle},\frac{u_2}{\langle(0.5,0.7);(0.1,0.2);(0.2,0.2)\rangle}\right\},$$

and we can write the 6-valued refined neutrosophic soft set (f_6,E) as consisting of the following collection of approximations:

$$\begin{aligned}(f_6,E)=&\\ \Bigg\{&\left(e_1,\left\{\frac{u_1}{\langle(0.5,0.4);(0.2,0.3);(0.4,0.3)\rangle},\frac{u_2}{\langle(0.7,0.4);(0.1,0.5);(0.5,0.1)\rangle}\right\}\right),\\ &\left(e_2,\left\{\frac{u_1}{\langle(0.3,0.5);(0.3,0.4);(0.5,0.4)\rangle},\frac{u_2}{\langle(0.3,0.3);(0.2,0.3);(0.4,0.8)\rangle}\right\}\right),\\ &\left(e_3,\left\{\frac{u_1}{\langle(0.6,0.3);(0.3,0.1);(0.2,0.4)\rangle},\frac{u_2}{\langle(0.5,0.7);(0.1,0.2);(0.2,0.2)\rangle}\right\}\right)\Bigg\}.\end{aligned}$$

Also we can represent the above set as shown in Table III.

Definition 11. *Let (f_n,A) and (g_n,B) be two n-valued refined neutrosophic soft sets over the common universe U. (f_n,A) is said to be n-valued refined neutrosophic soft subset of (g_n,B) if $A \subset B$; and $T_f^j(e)(x) \leq T_g^j(e)(x)$ where $j \in \{1,2,...,p\}$; $I_f^k(e)(x) \leq I_g^k(e)(x)$ where $k \in \{1,2,...,r\}$ and $F_f^l(e)(x) \geq F_g^l(e)(x)$ where $l \in \{1,2,...,s\}$; $\forall e \in A; x \in U$. We denote it by $(f_n,A) \subseteq (g_n,B)$. (f_n,A) is said to be n-valued refined neutrosophic soft super set of (g_n,B) if (g_n,B) is an n-valued refined neutrosophic soft subset of (f_n,A). We denote it by $(f_n,A) \supseteq (g_n,B)$.*

Definition 12. *Suppose $p = r$, the complement of an n-valued refined neutrosophic soft set (f_n,A) denoted by $(f_n;A)^c$ and is denoted as $(f_n,A)^c = (f_n^c, \rceil A)$; where $f_n^c : \rceil A \to P(U)$ is a mapping given by $f_n^c(x) =$ n-valued refined neutrosophic soft complement with $T_{f^c}^j(x) = F_f^j(x)$, $I_{f^c}^k(x) = I_f^k(x)$ and $F_{f^c}^j(x) = T_f^j(x)$ where $j \in \{1,2,...,p\}$ and $k \in \{1,2,...,r\}$.*

Example 4. *Consider the example 3 where*

$$f_5(e_1) = \left\{ \frac{u_1}{\langle 0.5;(0.2,0.3,0.4);0.4\rangle}, \frac{u_2}{\langle 0.7;(0.1,0.5,0.3);0.5\rangle} \right\},$$

$$f_5(e_2) = \left\{ \frac{u_1}{\langle 0.3;(0.3,0.4,0.5);0.5\rangle}, \frac{u_2}{\langle 0.3;(0.2,0.3,0.4);0.4\rangle} \right\},$$

$$f_5(e_3) = \left\{ \frac{u_1}{\langle 0.6;(0.3,0.1,0.2);0.2\rangle}, \frac{u_2}{\langle 0.5;(0.1,0.2,0.1);0.2\rangle} \right\}.$$

Then we can found the complement of (f_5,E) as the following 5-valued refined neutrosophic soft set $(f_5,E)^c$:

$$\begin{aligned}(f_5,E)^c = \Bigg\{ &\left(e_1, \left\{ \frac{u_1}{\langle 0.4;(0.2,0.3,0.4);0.5\rangle}, \frac{u_2}{\langle 0.5;(0.1,0.5,0.3);0.7\rangle} \right\}\right),\\ &\left(e_2, \left\{ \frac{u_1}{\langle 0.5;(0.3,0.4,0.5);0.3\rangle}, \frac{u_2}{\langle 0.4;(0.2,0.3,0.4);0.3\rangle} \right\}\right),\\ &\left(e_3, \left\{ \frac{u_1}{\langle 0.2;(0.3,0.1,0.2);0.6\rangle}, \frac{u_2}{\langle 0.2;(0.1,0.2,0.1);0.5\rangle} \right\}\right)\Bigg\}.\end{aligned}$$

Proposition 1. *If (f_n,A) is an n-valued refined neutrosophic soft set over U, and by using the n-valued refined neutrosophic soft complement then: $((f_n,A)^c)^c = (f_n,A)$,*

Proof.. The proof is straightforward from Definition 12. □

Definition 13. *Let (f_n,A) and (g_n,B) be two n-valued refined neutrosophic soft sets over the common universe U, we say that (f_n,A) and (g_n,B) are symmetric and denoted by $|(f_n,A)| \equiv |(g_n,B)|$ iff*

1) $|T_f| = |T_g|$,
2) $|I_f| = |I_g|$,
3) $|F_f| = |F_g|$.

Definition 14. *Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U . Then the union of (f_n,A) and (g_n,B) is denoted by '$(f_n,A) \cup (g_n,B)$' and is defined by $(f_n,A) \cup (g_n,B) = (h,C)$, where $C = A \cup B$ and the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of (h,C) are as follows:*

$\forall j \in \{1,2,...,p\}$

$$\begin{aligned}T_h^j(e)(m) &= T_f^j(e)(m); \quad \text{if } e \in A - B ;\\ &= T_g^j(e)(m); \quad \text{if } e \in B - A;\\ &= max(T_f^j(e)(m); T_g^j(e)(m)); \quad \text{if } e \in A \cap B.\end{aligned}$$

$\forall k \in \{1,2,...,r\}$

$$\begin{aligned}I_h^k(e)(m) &= I_f^k(e)(m); \quad \text{if } e \in A - B;\\ &= I_g^k(e)(m); \quad \text{if } e \in B - A;\\ &= \frac{I_f^k(e)(m) + I_g^k(e)(m))}{2}; \quad \text{if } e \in A \cap B.\end{aligned}$$

and $\forall l \in \{1,2,...,s\}$

$$\begin{aligned}F_h^l(e)(m) &= F_f^l(e)(m); \quad \text{if } e \in A - B;\\ &= F_g^l(e)(m); \quad \text{if } e \in B - A;\\ &= min(F_f^l(e)(m); F_g^l(e)(m)); \quad \text{if } e \in A \cap B.\end{aligned}$$

Example 5. *Let $U = \{u_1,u_2\}$ be a set of universe, $E = \{e_1,e_2,e_3\}$, let*

$$\begin{aligned}(f_4,E) = \Bigg\{ &\left(e_1, \left\{ \frac{u_1}{\langle 0.5;(0.2,0.3);0.4\rangle}, \frac{u_2}{\langle 0.7;(0.1,0.5);0.5\rangle} \right\}\right),\\ &\left(e_2, \left\{ \frac{u_1}{\langle 0.3;(0.3,0.4);0.5\rangle}, \frac{u_2}{\langle 0.3;(0.2,0.3);0.4\rangle} \right\}\right),\\ &\left(e_3, \left\{ \frac{u_1}{\langle 0.6;(0.3,0.1);0.2\rangle}, \frac{u_2}{\langle 0.5;(0.1,0.2);0.2\rangle} \right\}\right)\Bigg\}.\end{aligned}$$

and

$$\begin{aligned}(g_4,E) = \Bigg\{ &\left(e_1, \left\{ \frac{u_1}{\langle 0.3;(0.1,0.2);0.5\rangle}, \frac{u_2}{\langle 0.8;(0.4,0.6);0.3\rangle} \right\}\right),\\ &\left(e_2, \left\{ \frac{u_1}{\langle 0.5;(0.5,0.2);0.6\rangle}, \frac{u_2}{\langle 0.7;(0.6,0.2);0.3\rangle} \right\}\right),\\ &\left(e_3, \left\{ \frac{u_1}{\langle 0.5;(0.1,0.3);0.4\rangle}, \frac{u_2}{\langle 0.2;(0.4,0.2);0.4\rangle} \right\}\right)\Bigg\}.\end{aligned}$$

Then we can found the union of (f_4,E) and (g_4,E) as the following 4-valued refined neutrosophic soft set (h_4,E) :

$$\begin{aligned}(h_4,E) = \Bigg\{ &\left(e_1, \left\{ \frac{u_1}{\langle 0.5;(0.15,0.25);0.4\rangle}, \frac{u_2}{\langle 0.8;(0.25,0.55);0.3\rangle} \right\}\right),\\ &\left(e_2, \left\{ \frac{u_1}{\langle 0.5;(0.4,0.3);0.5\rangle}, \frac{u_2}{\langle 0.7;(0.4,0.25);0.3\rangle} \right\}\right),\\ &\left(e_3, \left\{ \frac{u_1}{\langle 0.6;(0.2,0.2);0.2\rangle}, \frac{u_2}{\langle 0.5;(0.25,0.2);0.2\rangle} \right\}\right)\Bigg\}.\end{aligned}$$

Proposition 2. *If (f_n,A), (g_n,B) and (h_n,C) are three symmetric n-valued refined neutrosophic soft sets over U, then*

1) $(f_n,A) \cup ((g_n,B)_t \cup (h_n,C)) = ((f_n,A) \cup (g_n,B)) \cup (h_n,C)$,
2) $(f_n,A) \cup (f_n,A) = (f_n,A)$.

Proof.. The proof is straightforward from Definition 14. □

Definition 15. *Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U . Then the intersection of (f_n,A) and (g_n,B) is denoted by '$(f_n,A) \cap (g_n,B)$' and is defined by $(f_n,A) \cap (g_n,B) = (d,C)$, where $C = A \cup B$ and the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of (d,C) are as follows: $\forall e \in C$*

$\forall j \in \{1,2,...,p\}$, $T_d^j(e)(m) = min(T_f^j(e)(m); T_g^j(e)(m))$;.

$\forall k \in \{1,2,...,r\}$, $I_d^k(e)(m) = \dfrac{I_f^k(e)(m) + I_g^k(e)(m))}{2}$

and $\forall l \in \{1,2,...,s\}$, $F_d^l(e)(m) = max(F_f^l(e)(m); F_g^l(e)(m))$.

Example 6. *Consider Example 5. Then we can found the intersection of (f_4,E) and (g_4,E) as the following 4-valued refined neutrosophic soft set (d_4,E) :*

$$(d_4,E) = \left\{ \left(e_1, \left\{ \frac{u_1}{\langle 0.3;(0.15,0.25);0.5\rangle}, \frac{u_2}{\langle 0.7;(0.25,0.55);0.5\rangle} \right\}\right), \left(e_2, \left\{ \frac{u_1}{\langle 0.3;(0.4,0.3);0.6\rangle}, \frac{u_2}{\langle 0.3;(0.4,0.25);0.4\rangle} \right\}\right), \left(e_3, \left\{ \frac{u_1}{\langle 0.5;(0.2,0.2);0.4\rangle}, \frac{u_2}{\langle 0.2;(0.25,0.2);0.4\rangle} \right\}\right) \right\}.$$

Proposition 3. *If (f_n,A), (g_n,B) and (h_n,C) are three symmetric n-valued refined neutrosophic soft sets over U, then*

1) $(f_n,A) \cap ((g_n,B)_t \cap (h_n,C)) = ((f_n,A) \cap (g_n,B)) \cap (h_n,C)$,
2) $(f_n,A) \cap (f_n,A) = (f_n,A)$.

Proof.. The proof is straightforward from Definition 15. □

Proposition 4. *If (f_n,A), (g_n,B) and (h_n,C) are three symmetric n-valued refined neutrosophic soft sets over U, then*

1) $(f_n,A) \cup ((g_n,B) \cap (h_n,C)) = ((f_n,A) \cup (g_n,B)) \cap ((f_n,A) \cup (h_n,C))$,
2) $(f_n,A) \cap ((g_n,B) \cup (h_n,C)) = ((f_n,A) \cap (g_n,B)) \cup ((f_n,A) \cap (h_n,C))$.

Proof.. The proof is straightforward from Definitions 15 and 14. □

Definition 16. *Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U. Then the 'AND' operation on them is denoted by '$(f_n,A) \bigvee (g_n,B)$' and is defined by $(f_n,A) \bigvee (g_n,B) = (q_n, A \times B)$, where the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of $(q_n, A \times B)$ are as follows: $\forall \alpha \in A, \forall \beta \in B$.*

$\forall j \in \{1,2,...,p\}$, $T_q^j(\alpha,\beta)(m) = min(T_f^j(\alpha)(m); T_g^j(\beta)(m))$;

$\forall k \in \{1,2,...,r\}$, $I_q^k(\alpha,\beta)(m) = \dfrac{I_f^k(\alpha)(m) + I_g^k(\beta)(m))}{2}$ *and*

$\forall l \in \{1,2,...,s\}$, $F_q^l(\alpha,\beta)(m) = max(F_f^l(\alpha)(m); F_g^l(\beta)(m))$.

Definition 17. *Let (f_n,A) and (g_n,B) be two symmetric n-valued refined neutrosophic soft sets over the common universe U. Then the 'OR' operation on them is denoted by '$(f_n,A) \bigwedge (g_n,B)$' and is defined by $(f_n,A) \bigwedge (g_n,B) = (q_n, A \times B)$, where the p-truth-memberships, r-indeterminacy-memberships and s-falsity-memberships of $(q_n, A \times B)$ are as follows: $\forall \alpha \in A, \forall \beta \in B$.*

$\forall j \in \{1,2,...,p\}$, $T_q^j(\alpha,\beta)(m) = max(T_f^j(\alpha)(m); T_g^j(\beta)(m))$;

$\forall k \in \{1,2,...,r\}$, $I_q^k(\alpha,\beta)(m) = \dfrac{I_f^k(\alpha)(m) + I_g^k(\beta)(m))}{2}$ *and*

$\forall l \in \{1,2,...,s\}$, $F_q^l(\alpha,\beta)(m) = min(F_f^l(\alpha)(m); F_g^l(\beta)(m))$.

Proposition 5. *If (f_n,A) and (g_n,B) are two symmetric n-valued refined neutrosophic soft sets over U, then*

1) $((f_n,A) \wedge (g_n,B))^c = (f_n,A)^c \vee (g_n,B)^c$
2) $((f_n,A) \vee (g_n,B))^c = (f_n,A)^c \wedge (g_n,B)^c$

Proof.. The proof is straightforward from Definitions 16, 17 and 12. □

Proposition 6. *If (f_n,A), (g_n,B) and (h_n,C) are three symmetric n-valued refined neutrosophic soft sets over U, then*

1) $(f_n,A) \wedge ((g_n,B) \wedge (h_n,C)) = ((f_n,A) \wedge (g_n,B)) \wedge (h_n,C)$,
2) $(f_n,A) \vee ((g_n,B) \vee (h_n,C)) = ((f_n,A) \vee (g_n,B)) \vee (h_n,C)$,
3) $(f_n,A) \vee ((g_n,B) \wedge (h_n,C)) = ((f_n,A) \vee (g_n,B)) \wedge ((f_n,A) \vee (h_n,C))$,
4) $(f_n,A) \wedge ((g_n,B) \vee (h_n,C)) = ((f_n,A) \wedge (g_n,B)) \vee ((f_n,A) \wedge (h_n,C))$.

Proof.. The proof is straightforward from Definitions 16 and 17. □

IV. CONCLUSION

In this paper we have introduced the concept of n-valued refined neutrosophic soft set and studied some of its properties. The complement, union, intersection, OR and AND operations have been defined on the n-Valued refined neutrosophic soft set. Future possible research of the authors will be to give an application of this theory in solving a decision making problem and medical digenesis also the authors can extend this n-valued refined neutrosophic soft set to to time-refined-neutrosophic soft set.

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TABLE I
4-VALUED REFINED NEUTROSOPHIC SOFT SET (f_4, E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T	0.5	0.3	0.6	0.7	0.3	0.5
Un	0.2	0.3	0.3	0.1	0.2	0.1
C	0.3	0.4	0.1	0.5	0.3	0.2
F	0.4	0.5	0.2	0.5	0.4	0.2

TABLE II
5-VALUED REFINED NEUTROSOPHIC SOFT SET (f_5, E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T	0.5	0.3	0.6	0.7	0.3	0.5
Un	0.2	0.3	0.3	0.1	0.2	0.1
C	0.3	0.4	0.1	0.5	0.3	0.2
G	0.4	0.5	0.2	0.3	0.4	0.1
F	0.4	0.5	0.2	0.5	0.4	0.2

TABLE III
6-VALUED REFINED NEUTROSOPHIC SOFT SET (f_6, E)

	(u_1,e_1)	(u_1,e_2)	(u_1,e_3)	(u_2,e_1)	(u_2,e_2)	(u_2,e_3)
T_A	0.5	0.3	0.6	0.7	0.3	0.5
T_R	0.4	0.5	0.3	0.4	0.3	0.7
I_A	0.2	0.3	0.3	0.1	0.2	0.1
I_R	0.3	0.4	0.1	0.5	0.3	0.2
F_A	0.4	0.5	0.2	0.5	0.4	0.2
F_R	0.3	0.4	0.4	0.1	0.8	0.2

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